Unit 1

Chapter 1.1 Rational Numbers

Operations Research Analyst

Sports Event Organiser

Rational Numbers

A rational number is defined as a number that can be expressed in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

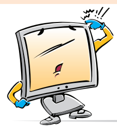
Home Page

We often use rational expressions to represent many things in our day-to-day life. For example, a music composer makes musical notes that vary in length (these can be whole (1), half (½) or quarter (¼) notes). Rational numbers also provide the foundation upon which elementary algebraic operations are based. They are also applied in various areas such as measurement, probability, coordinate systems, graphing, etc.

So, let's learn more about rational numbers in detail, including the properties of operations on different types of numbers such as whole numbers, integers and rational numbers. Also, we will learn about the representation of rational numbers on the number line and how to find rational numbers between two given rational numbers.

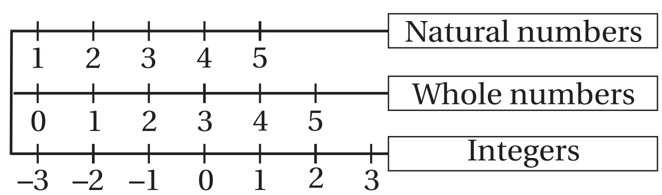
Rational Numbers

- **Properties of Rational Numbers**
  - Closure
  - Commutative
  - Associative
  - Distributive
  - Additive identity
  - Multiplicative identitiy
  - Additive inverse
  - Multiplicative inverse
- **Representation of Rational Numbers on the Number Line**
- **Rational Numbers between Two Rational Numbers**
  - LCM method
  - Average method



Retrieve

- **Closure:** If we combine two elements from a set of numbers by using an operation (+, –, ×, ÷) and the result also falls in the same set, we say that the two elements are closed under that operation.
- **Commutative:** An operation is said to be commutative if a change in the order of the numbers does not change the result, i.e., $a * b = b * a$, where a and b are any numbers and $*$ can be (+, –, ×, ÷).
- **Associative:** An operation is said to be associative if a change in the grouping of the numbers does not change the result, i.e., $a * (b * c) = (a * b) * c$, where a, b and c are any numbers and $*$ can be (+, –, ×, ÷).
- **Distributive:** The product of a number with the sum or difference of any two numbers is equal to the sum or difference of the individual products of the number and the addends, i.e., $a \times (b * c) = a \times b * a \times c$, where a, b and c are any numbers and $*$ can be (+, –).
- **Representation of numbers on the number line**



1. Verify $a \times (b + c) = a \times b + a \times c$ for the given values of a, b and c.

   (a) $a = -3$, $b = 2$ and $c = 5$

   (b) $a = 13$, $b = -22$ and $c = 7$

   ___

   ___

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2. Simplify $\frac{11}{3} \times \frac{1}{33} + \frac{9}{2} \times \frac{2}{81}$.

   ___

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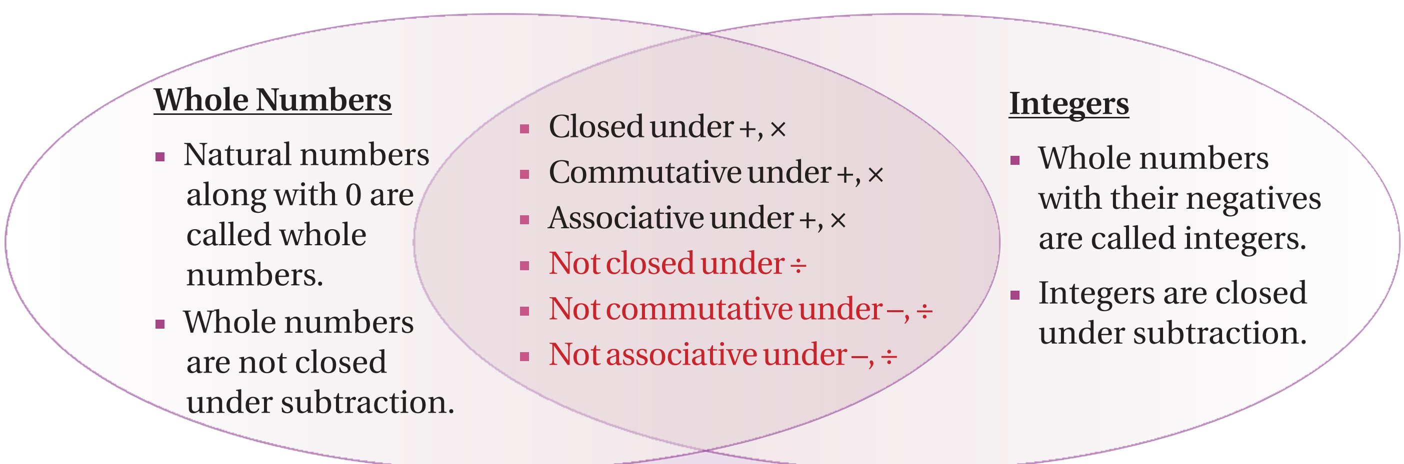
Upload Rational Numbers

A rational number is defined as a number that can be expressed in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$. The set of rational numbers is denoted by Q and it contains fractions, decimals and integers.

Properties of Rational Numbers

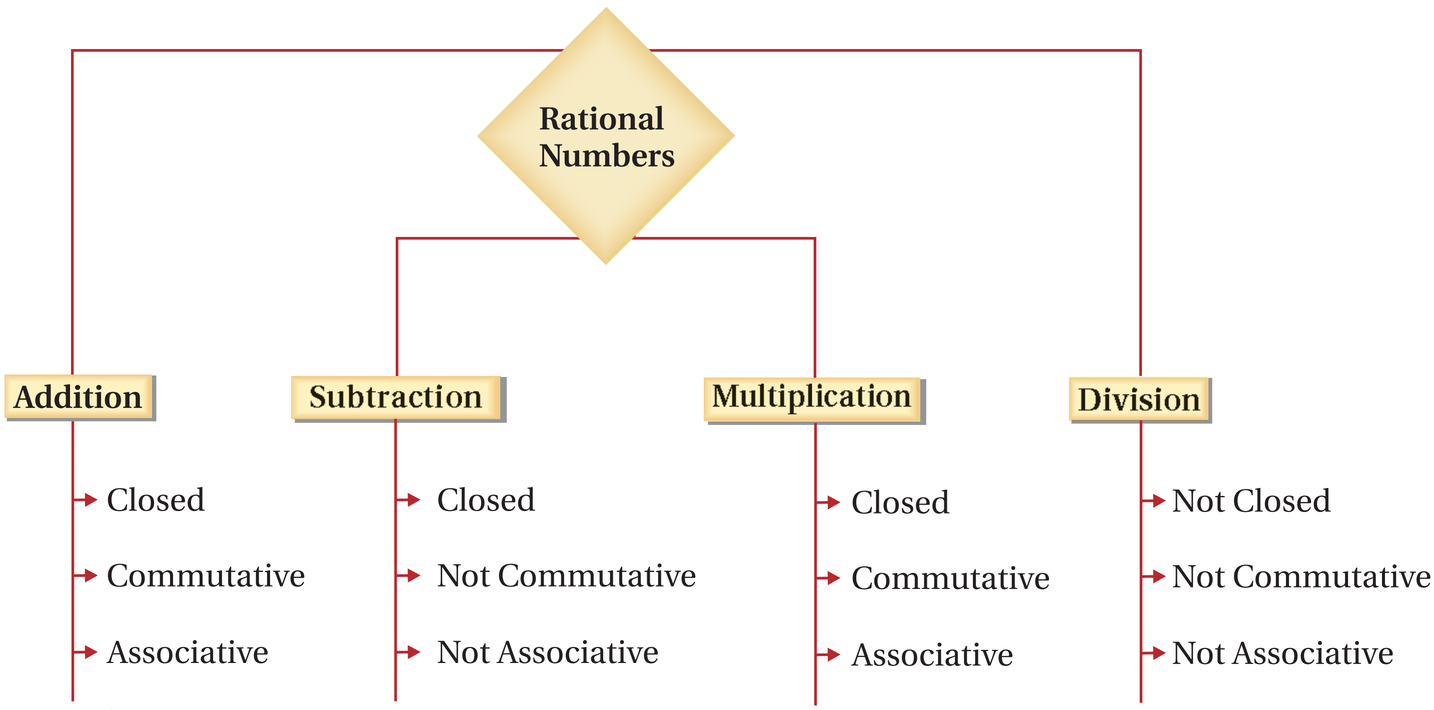
Before learning about the properties of rational numbers, let's compare the properties of whole numbers and integers through the Venn diagram given below.



Example

	Whole Numbers			Integers		
Operation	**Closure**	**Commutative**	**Associative**	**Closure**	**Commutative**	**Associative**
+	$4 + 8 = 12$ *(closed)*	$2 + 7 = 7 + 2$ *(commutative)*	$3 + (7 + 4) = (3 + 7) + 4$ *(associative)*	$-7 + 5 = -2$ *(closed)*	$-8 + 5 = 5 + (-8) = -3$ *(commutative)*	$-6 + \{5 + (-4)\} = (-6 + 5) + (-4)$ *(associative)*
–	$7 - 9 = -2$ *(not closed)*	$5 - 7 \neq 7 - 2$ *(not commutative)*	$8 - (4 - 6) \neq (8 - 4) - 6$ *(not associative)*	$-4 - 2 = -6$ *(closed)*	$7 - (-6) \neq -6 - 7$ *(not commutative)*	$3 - (5 - 9) \neq (3 - 5) - 9$ *(not associative)*
×	$5 \times 4 = 20$ *(closed)*	$12 \times 3 = 3 \times 12$ *(commutative)*	$2 \times (7 \times 5) = (2 \times 7) \times 5$ *(associative)*	$-3 \times 2 = -6$ *(closed)*	$-4 \times 8 = 8 \times (-4)$ *(commutative)*	$6 \times \{(-4) \times (-2)\} = \{6 \times (-4)\} \times (-2)$ *(associative)*
÷	$4 \div 5 = \frac{4}{5}$ *(not closed)*	$11 \div 3 \neq 3 \div 11$ *(not commutative)*	$25 \div (5 \div 15) \neq (25 \div 5) \div 15$ *(not associative)*	$-2 \div 9 = \frac{-2}{9}$ *(not closed)*	$-7 \div 11 \neq 11 \div (-7)$ *(not commutative)*	$(-6) \div \{(-3) \div (-12)\} \neq \{(-6) \div (-3)\} \div (-12)$ *(not associative)*

In earlier classes, you have studied about rational numbers. Let's now study about their properties under different operations $(+, -, \times, \div)$.



Examples:

	Addition	Subtraction	Multiplication	Division
Closure	$\frac{2}{3}+\frac{1}{2}=\frac{7}{6}$ *(closed)*	$\frac{2}{3}-\frac{1}{2}=\frac{1}{6}$ *(closed)*	$\frac{2}{3}\times\frac{1}{2}=\frac{1}{3}$ *(closed)*	$\frac{2}{3}\div 0 =$ Not defined *(not closed)*
Commutative	$\frac{2}{3}+\frac{1}{2}=\frac{1}{2}+\frac{2}{3}$ *(commutative)*	$\frac{2}{3}-\frac{1}{2}\neq\frac{1}{2}-\frac{2}{3}$ *(not commutative)*	$\frac{2}{3}\times\frac{1}{2}=\frac{1}{2}\times\frac{2}{3}$ *(commutative)*	$\frac{2}{3}\div\frac{1}{2}\neq\frac{1}{2}\div\frac{2}{3}$ *(not commutative)*
Associative	$\frac{2}{3}+\left(\frac{1}{2}+\frac{3}{7}\right)=\left(\frac{2}{3}+\frac{1}{2}\right)+\frac{3}{7}$ *(associative)*	$\frac{2}{3}-\left(\frac{1}{2}-\frac{3}{7}\right)\neq\left(\frac{2}{3}-\frac{1}{2}\right)-\frac{3}{7}$ *(not associative)*	$\frac{2}{3}\times\left(\frac{1}{2}\times\frac{3}{7}\right)=\left(\frac{2}{3}\times\frac{1}{2}\right)\times\frac{3}{7}$ *(associative)*	$\frac{2}{3}\div\left(\frac{1}{2}\div\frac{3}{7}\right)\neq\left(\frac{2}{3}\div\frac{1}{2}\right)\div\frac{3}{7}$ *(not associative)*

If zero is excluded, then all rational numbers are closed under division.

Let us learn about some more properties of rational numbers as given below.

Distributivity of multiplication over addition and subtraction

for any rational numbers a, b and c

$a \times (b + c) = (a \times b) + (a \times c)$

Example:

$\frac{2}{3} \times \left(3 + \frac{2}{5}\right) = \left(\frac{2}{3} \times 3\right) + \left(\frac{2}{3} \times \frac{2}{5}\right) = \frac{34}{15}$

$a \times (b - c) = (a \times b) - (a \times c)$

Example:

$\frac{1}{5} \times \left(\frac{3}{4} - \frac{2}{3}\right) = \left(\frac{1}{5} \times \frac{3}{4}\right) - \left(\frac{1}{5} \times \frac{2}{3}\right) = \frac{1}{60}$

For any rational number $\frac{a}{b}, b \neq 0$ we have:

1

Additive Identity

The property of Additive Identity states that the sum of the number and zero (0) yields the same number.

In other words, $\mathbf{0} + \frac{a}{b} = \frac{a}{b} = \frac{a}{b} + \mathbf{0}$

Example: $\mathbf{0} + \frac{2}{5} = \frac{2}{5} = \frac{2}{5} + \mathbf{0}$

Conclusion: **Zero (0)** is called the additive identity of rational numbers.

2

Multiplicative Identity

The property of Multiplicative Identity states that the product of any number and one (1) is the number itself.

In other words, $\mathbf{1} \times \frac{a}{b} = \frac{a}{b} = \frac{a}{b} \times \mathbf{1}$

Example: $\mathbf{1} \times \left(\frac{-5}{6}\right) = \frac{-5}{6} = \frac{-5}{6} \times \mathbf{1}$

Conclusion: **One (1)** is the multiplicative identity of rational numbers.

3

Additive Inverse

Additive Inverse of the number $\frac{a}{b}$ will be a number which when added to $\frac{a}{b}$ yields zero.

In other words, $\frac{a}{b} + \left(\frac{-a}{b}\right) = 0 = \left(\frac{-a}{b}\right) + \frac{a}{b}$

Example: $\frac{1}{3} + \left(\frac{-1}{3}\right) = 0 = \left(\frac{-1}{3}\right) + \frac{1}{3}$

Conclusion: $\frac{-a}{b}$ is the additive inverse of $\frac{a}{b}$ and vice versa. It is also known as the **negative** of the number.

4

Multiplicative Inverse

The Multiplicative Inverse of a number $\frac{a}{b}$ will be a number which when multiplied by $\frac{a}{b}$ yields 1.

In other words, $\frac{a}{b} \times \frac{b}{a} = 1 = \frac{b}{a} \times \frac{a}{b}$

Example: $\frac{7}{3} \times \frac{3}{7} = 1 = \frac{3}{7} \times \frac{7}{3}$

Conclusion: $\frac{b}{a}$ is the multiplicative inverse of $\frac{a}{b}$ and vice versa. It is also known as the **reciprocal** of the number.

The above properties also hold true for whole numbers and integers.

- Additive Identity: '0'
- Mulitplicative Identity: '1'
- Additive Inverse: 'negative'
- Multiplicative Inverse: 'reciprocal'
- Negative of a negative number is positive.

The addition, multiplication, subtraction or division of two rational numbers results in a rational number.

✓ $\frac{2}{3}+\frac{1}{2}=\frac{7}{6}$ ✓ $\frac{2}{3}\times\frac{1}{2}=\frac{1}{3}$ ✓ $\frac{2}{3}-\frac{1}{2}=\frac{1}{6}$ ✓ $\frac{2}{3}\div\frac{1}{2}=\frac{4}{3}$

Example 1: Find $\left(\frac{9}{16}\times\frac{4}{12}\right)+\left(\frac{9}{16}\times\frac{-3}{9}\right)$ using distributive property of rational numbers.

Solution:

$$\left(\frac{9}{16}\times\frac{4}{12}\right)+\left(\frac{9}{16}\times\frac{-3}{9}\right)=\frac{9}{16}\times\left(\frac{4}{12}+\frac{-3}{9}\right)$$

Using $(a \times b) + (a \times c) = a \times (b + c)$

$$=\frac{9}{16}\times\left(\frac{12-12}{36}\right)$$

$$=\frac{9}{16}\times\frac{0}{36}$$

$$=0$$

Example 2: Using appropriate properties, find $\frac{-2}{3}\times\frac{3}{5}+\frac{5}{2}-\frac{3}{5}\times\frac{1}{6}$.

Solution: We can write

$$\frac{-2}{3}\times\frac{3}{5}+\frac{5}{2}-\frac{3}{5}\times\frac{1}{6}=\left(\frac{-2}{3}\times\frac{3}{5}\right)+\frac{5}{2}-\left(\frac{3}{5}\times\frac{1}{6}\right)$$

$$=\frac{5}{2}+\left(\frac{-2}{3}\times\frac{3}{5}\right)-\left(\frac{3}{5}\times\frac{1}{6}\right)$$ [Using commutative property of addition]

$$=\frac{5}{2}+\left(\frac{3}{5}\times\frac{-2}{3}\right)-\left(\frac{3}{5}\times\frac{1}{6}\right)$$ [Using commutative property of multiplication]

$$=\frac{5}{2}+\frac{3}{5}\times\left(\frac{-2}{3}-\frac{1}{6}\right)$$ [Using distributive property of multiplication over subtraction]

$$=\frac{5}{2}+\frac{3}{5}\times\left(\frac{-4-1}{6}\right)$$

$$=\frac{5}{2}+\frac{3}{5}\times\frac{-5}{6}$$

$$=\frac{5}{2}-\frac{1}{2}$$

$$=\frac{4}{2}=2$$

Observe the problem
Read the signs
Decide which operation to do first
Execute the rule of order
Relax, you are done

Example 3: If the statements given below are true for all rational numbers, find x in each case.

(a) $\frac{42}{5} \times x = \frac{42}{5}$ (b) $\frac{21}{5} \times \left(\frac{51}{2} - x\right) = \frac{21}{5} \times \frac{51}{2} - \frac{21}{5} \times \frac{1}{8}$

Solution: (a) $\frac{42}{5} \times x = \frac{42}{5}$ ...(i)

By multiplicative identity property, we know that the product of any number and 1 is the number itself. ....(ii)

Therefore, from (i) and (ii) , we get

$x = 1$

(b) $\frac{21}{5} \times \left(\frac{51}{2} - x\right) = \frac{21}{5} \times \frac{51}{2} - \frac{21}{5} \times \frac{1}{8}$ ...(i)

By distributivity of multiplication over subtraction, we know that
$a \times (b - c) = (a \times b) - (a \times c)$, where a, b and c are any rational numbers. ...(ii)

Therefore, from (i) and (ii), we get

$x = \frac{1}{8}$

Example 4: The sum of two rational numbers is $\frac{27}{9}$. If one of them is $\frac{1}{3}$, find the other.

Solution: Let the other number be x. Then

$$x + \frac{1}{3} = \frac{27}{9}$$

$$\Rightarrow x + 0 = \frac{27}{9} + \left(\text{additive inverse of } \frac{1}{3}\right) \qquad \left\{\because \frac{1}{3} + \text{additive inverse of } \left(\frac{1}{3}\right) = \frac{1}{3} + \left(\frac{-1}{3}\right) = 0\right\}$$

$$\Rightarrow x = \frac{27}{9} + \left(\frac{-1}{3}\right) = \frac{27-3}{9}$$

$$\Rightarrow x = \frac{24}{9} = \frac{8}{3}$$

Therefore, the required number is $\frac{8}{3}$.

Example 5: Riya ate $\left(\frac{11}{50}\right)^{th}$ part of a cake on the first day and $\left(\frac{1}{25}\right)^{th}$ part on the second day. On the third day, she and her two friends together ate $\left(\frac{11}{25}\right)^{th}$ part of the cake. On the fourth day, Riya's mother and brother decided to eat half of the total cake but the left over part of the cake was not sufficient. Explain why it happened?

Solution: On the first day, the part of the cake eaten by Riya = $\frac{11}{50}$

On the second day, the part eaten by her = $\frac{1}{25}$

On the third day, the part eaten by Riya and her two friends = $\frac{11}{25}$

So, the total part of the cake eaten in all = $\frac{11}{50}+\frac{1}{25}+\frac{11}{25}$

$$= \frac{11+2+22}{50} = \frac{35}{50} = \frac{7}{10}$$

This clearly shows that the cake left was $\left(\frac{3}{10}\right)^{th}$ part of the total quantity which is less than half of the total cake. So, the quantity left was less than half of the total cake.

Apps

A cycling championship is going to be organised in your school. $\left(\frac{5}{18}\right)^{th}$ of the total number of students in the school have been selected for it. The sponsor for this championship has agreed to give the following items for each participant:

Name of the item	T-shirt	Cap	Water bottle	Shoes
Cost per item	₹ $\frac{143}{2}$	₹ $\frac{522}{9}$	₹ $\frac{121}{2}$	₹ $\frac{910}{7}$

Find out how much it will cost the sponsor to supply

(i) t-shirts and caps

(ii) water bottles and shoes

for all the participants, if the total strength of the school is 540. Write the mathematical expression and state the property of rational numbers that can be used to calculate the total cost of the items provided by the sponsor to all the participants.

Sports Events Organiser

You might have watched the Commonwealth Games, 2010, held in Delhi. It was a mega multi-sport event that showcased 17 different types of sports and brought together sportspeople from several countries. All sports played at national or international levels require funding and sponsorship to support the expenses involved, organising and hosting the events, etc.

Planning and organising such events involves the use of numbers in planning the number of seats to be arranged, time duration, number of participants, etc. Financial planning for the event also requires a proper understanding of numbers. In such cases rational numbers play an important role.

A sports event organiser is a person who organises such sports events and arranges funds and facilities required for them. Also, the promotional activities for these sports events are handled by the sports event organiser. They earn profits from the event and the satisfaction of interacting with sportspersons and a large and varied audience, and, of course, the thrill of watching a world-class event.

Absolute value of a rational number

The absolute value of a rational number is its distance from zero on the number line. It is basically the positive value of a rational number. For example, the absolute value of $\frac{1}{7}$ is $\frac{1}{7}$ and the absolute value of $\frac{-1}{7}$ is also $\frac{1}{7}$ because each number is at a distance of $\frac{1}{7}$ units from 0.

Processing—I

1. Complete the given table.

Properties / Numbers	Closed under			
	Addition	Subtraction	Multiplication	Division
Natural numbers	Yes			No
Whole numbers		No	Yes	
Integers		Yes		No
Rational numbers	Yes		Yes	

2. Write the additive inverse of $\frac{-7}{-9}$.

3. Find the multiplicative inverse of $\frac{2}{3} \times \frac{-1}{2}$.

4. Which property allows us to compute $\frac{3}{2} + \left(\frac{-1}{3} + 6\right)$ as $\left(\frac{3}{2} - \frac{1}{3}\right) + 6$?

5. If the statements given below are true for all rational numbers, find x in each case.

   (a) $\frac{-4}{3} \times \left\{\frac{1}{5} + \left(\frac{-8}{3}\right)\right\} = \frac{-4}{3} \times \frac{1}{5} + x \times \left(\frac{-8}{3}\right)$ (b) $\frac{-5}{9} \div \frac{17}{4} = \frac{-5}{9} \times x$

6. From a 9 m long cloth, three pieces of cloth of lengths $2\frac{2}{5}$ m, $3\frac{1}{5}$ m and $2\frac{1}{5}$ m respectively are cut off. What is the length of the remaining cloth?

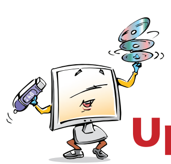
7. Using suitable properties, find $\frac{-1}{2} \times \frac{4}{5} \times \frac{15}{8} \times \frac{4}{7}$.

8. Using properties of rational numbers, evaluate

   (a) $\frac{12}{7} \times \frac{1}{4} + \frac{1}{20} - \frac{2}{5} \times \frac{35}{8}$ (b) $\frac{2}{5} \times \frac{-3}{7} - \frac{1}{14} - \frac{3}{7} \times \frac{3}{5}$

9. Verify commutativity of addition of rational numbers for each of the following pairs of rational numbers.

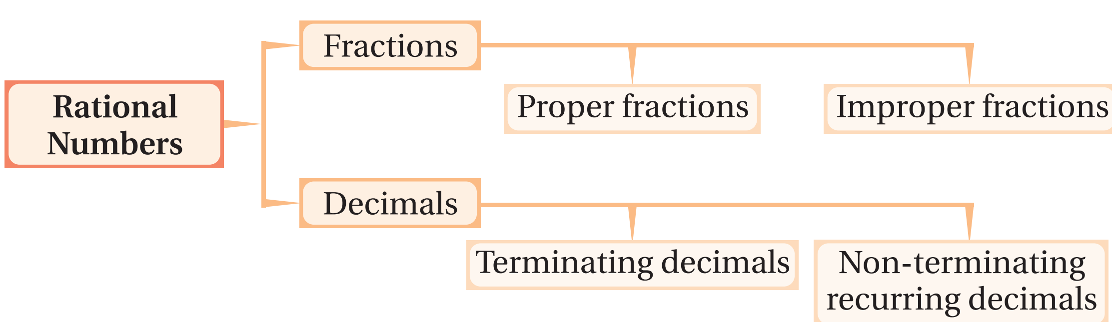
   (a) $\frac{-21}{5}$ and $\frac{2}{4}$ (b) -8 and $\frac{8}{3}$



Upload Representation of Rational Numbers on the Number Line

In earlier classes, you have learnt about representing natural numbers, whole numbers and integers on the number line.

Rational numbers can also be represented on the number line. The set of rational numbers, denoted by Q, consists of fractions and decimals as shown below.



Terminating decimals are decimal numbers that have a finite number of digits after the decimal point.

Non-terminating recurring decimals are decimal numbers in which the sequence of digits after the decimal point appears repetitively infinitely.

The number line helps in comparing two or more rational numbers and arranging them in ascending or descending order too. Let's check out how rational numbers can be represented on the number line. This is done by looking at the numerator and denominator of rational numbers in the following manner:

Denominator : Number of equal parts into which the unit is to be divided

Numerator : Number of parts that are to be taken into consideration

Consider the given number line:

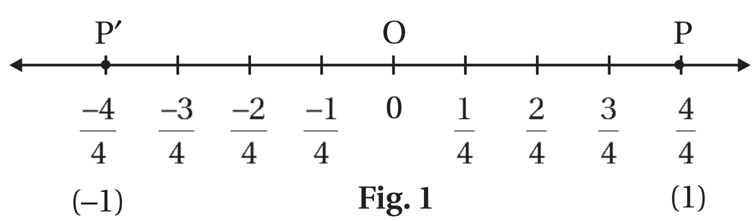


Fig. 1

Here, the points P and P′ are at **equal distance** from O but in **opposite directions** such that OP = 1 and OP′ = –1, where OP and OP′ are the first unit lengths on the right and left of O, respectively.

Now, consider the following examples to learn the representation of rational numbers (fractions and decimals) on the number line.

Example 1: Represent $\frac{4}{9}$ and $\frac{-4}{9}$ on the number line.

This is the case of proper fractions.

Solution: Let us first consider the representation of $\frac{4}{9}$ (**positive fraction**) on the number line.

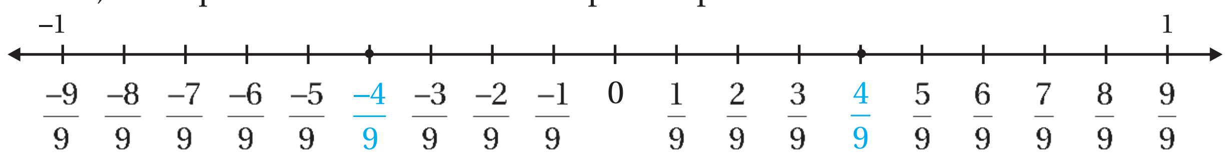
The given fraction is a proper fraction and a proper fraction always lies between 0 and 1.

Here, the denominator is 9. Therefore, the first unit length of the number line on the right side of O should be divided equally into 9 parts.

The numerator is 4. Therefore, out of 9 divisions on the number line, 4 parts should be considered. The number will therefore lie on the fourth part on the number line.

Similarly, $\frac{-4}{9}$ (**negative fraction**) can also be represented on the number line in the same way on the left side of O.

Hence, the required number line with the plotted points will be :



> Can you see that $\frac{9}{9}$ is same as 1 and $\frac{-9}{9}$ is same as –1 on the number line?

Example 2: Represent $\frac{14}{3}$ and $\frac{-14}{3}$ on the number line. Are these points equidistant from O?

Solution: Consider $\frac{14}{3}$.

> This is the case of improper fractions.

To represent $\frac{14}{3}$ on the number line, follow the steps given below:

(a) Now, $\frac{14}{3} = 4 + \frac{2}{3}$. This indicates that the given rational number should lie between the integers 4 and 5.

> To find two integers between which the given fractions lie, we need to convert the improper fraction to mixed fractions.

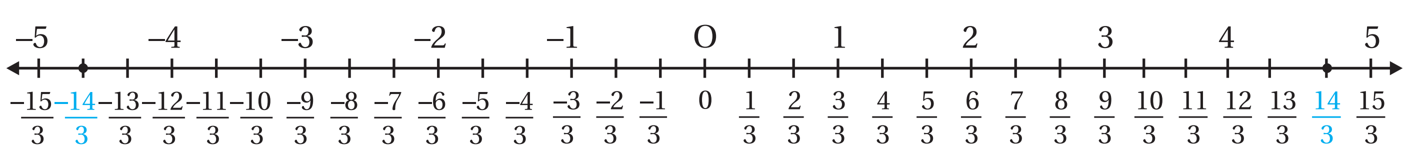
(b) Show 5 unit lengths on both sides of O on the number line.

(c) Now, divide the distance between the fourth and fifth units into three equal parts (3 being the denominator of $\frac{2}{3}$).

(d) Consider the second part out of these three parts (2 being the numerator of $\frac{2}{3}$) and locate the point on the second division.

(e) Similarly, $\frac{-14}{3}$ can be represented on the number line on the left side of O.

Hence, the required number line with the plotted points will be:

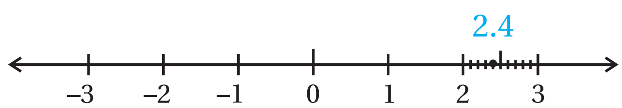


> In this example, the given number is an improper fraction that is first converted into a mixed fraction. Can you tell why?
> Because it helps to decide the two integers between which the fraction lies.

Example 3: Represent 2.4 on the number line.

Solution: Here, 2.4 is a terminating decimal number.

Divide the unit length between 2 and 3 into 10 equal parts and represent 2.4 as the fourth part from 2, as shown below.



Example 4: Represent $3.\overline{6}$ on the number line.

Solution: Here, $3.\overline{6}$ is a non-terminating recurring decimal number.

First convert the non-terminating recurring decimal into fraction form.

Let $x = 3.\overline{6} = 3.66666......$ ...(i)

Multiply both sides of (i) by 10.

$\Rightarrow 10x = 36.6666.......$ ...(ii)

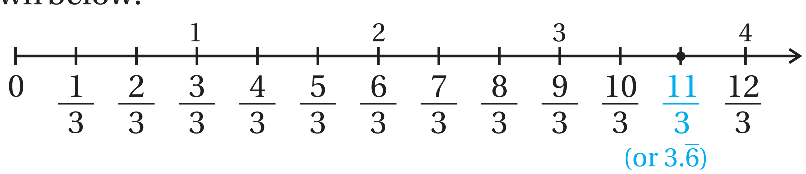
Subtract (i) from (ii) we get

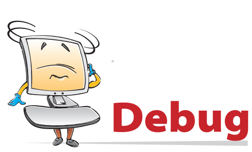
$10x - x = (36.6666......) - (3.66666......)$

$\Rightarrow 9x = 33$

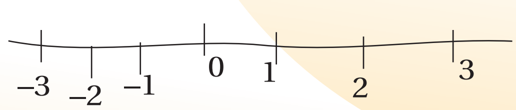
$\Rightarrow x = \frac{33}{9} = \frac{11}{3}$

Following the same method as explained in example 1, we get the required number line as shown below:





Incorrect



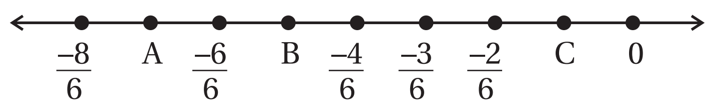
- Line is not straight.
- Unequal division of units on the number line

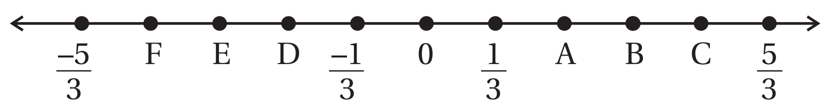
Correct

- Draw a straight line.
- Place the numbers neatly at equal distances on a number line.

Processing—II

1. Identify the missing rational numbers on the given number line.

(a) 

(b) 

2. Simplify the following and plot the result on the number line.

(a) $\frac{1}{3}+\frac{1}{5}$ (b) $\frac{4}{6}\times\frac{3}{2}$ (c) $\frac{1}{2}+\frac{1}{3}\times 6$ (d) $-3\times\frac{2}{5}+\frac{1}{5}$

3. Represent the following points on the number line.

(a) $\frac{2}{3}$ (b) $-3\frac{2}{5}$

(c) $0.\overline{5}$ (d) 3.5

4. Represent and mark the following points on the number line.

$A=\frac{5}{3}$, $B=\frac{-1}{3}$, $C=2\frac{2}{3}$ and $D=-4\frac{1}{3}$

Identify the given numbers as terminating or non-terminating recurring decimals and apply the appropriate method to represent them on the number line.

Apps

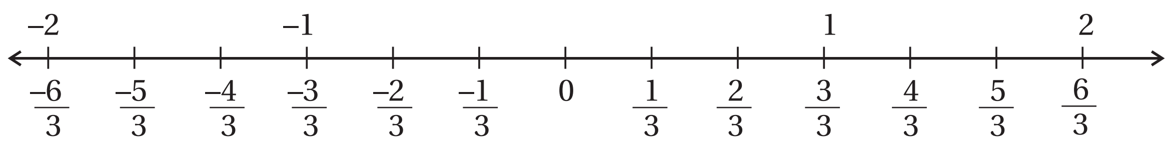
Get into pairs. Each one of you will draw a number line on a sheet of paper and think of any 3 numeric values (could be integers, whole numbers, decimals, fractions, etc.). Exchange these numbers with each other. Represent the numbers given by your partner on your number line. Are you able to represent each and every number on the same number line? Write your conclusion and the changes you needed to make to the number line while representing the three types of numbers.

Again, think of any rational expression (e.g., $3\times\frac{4}{5}-2.5$). Exchange that rational expression with each other.

Solve it and represent the result on another number line. Label that point with the initial of your name. Also, mark the result of your partner's rational expression on the same number line and label it with the initial of your partner's name. Compare both the rational numbers and circle the greater one.

Upload Rational Numbers between Two Rational Numbers

Consider the following rational numbers on the number line.



Can you tell the number of rational numbers between any two rational numbers by looking at the number line? Do you think there can be more rational numbers than those shown on the given number line? Let us check out how to find rational numbers between two given rational numbers.

There are two methods to find the rational numbers between any two rational numbers.

LCM method and **Average method**

Lets us learn more about these methods with the help of the following examples.

LCM method

Example: To find rational numbers between $\frac{4}{5}$ and $\frac{7}{5}$

Since both the given rational numbers have the same denominators, we will consider the numbers between the numerators 4 and 7.

Apparently, $\frac{5}{5}$ and $\frac{6}{5}$ lie between the given rational numbers.

To find more rational numbers between the given numbers, we have to find their equivalents.
For this, we multiply the numerator and denominator of the rational numbers by the same number, say 2. Then we get,

$\frac{4\times 2}{5\times 2} = \frac{8}{10}$ and $\frac{7\times 2}{5\times 2} = \frac{14}{10}$

Now, the rational numbers that lie between $\frac{8}{10}$ and $\frac{14}{10}$

are $\frac{9}{10}, \frac{10}{10}, \frac{11}{10}, \frac{12}{10}, \frac{13}{10}$.

Therefore, we have

$\frac{4}{5} < \frac{9}{10} < \frac{10}{10} < \frac{11}{10} < \frac{12}{10} < \frac{13}{10} < \frac{7}{5}$

Average method

Example: $\frac{3}{5}$ and $\frac{3}{4}$

Here, both the rational numbers have different denominators. We will find their mean or average.

Mean of $\frac{3}{5}$ and $\frac{3}{4}$

$= \left(\frac{3}{5} + \frac{3}{4}\right) \div 2$

$= \left(\frac{12 + 15}{20}\right) \div 2$

$= \frac{27}{20} \div 2 = \frac{27}{40}$

$\frac{27}{40}$ lies between $\frac{3}{5}$ and $\frac{3}{4}$.

Again, $\left(\frac{3}{5} + \frac{27}{40}\right) \div 2$

$= \frac{24 + 27}{40} \div 2$

$= \frac{51}{40} \div 2 = \frac{51}{80}$

$\frac{51}{80}$ lies between $\frac{3}{5}$ and $\frac{27}{40}$

and so on.

Thus, we can find an infinite number of rational numbers between any two rational numbers. This is known as the 'Density Property' of rational numbers.

OPERATIONS RESEARCH ANALYST

We often come across situations in daily life where we have to face complex problems along with limitations such as lack of time, shortage of resources, etc. In such situations, we need to analyse and interpret the whole situation in order to provide alternate solutions and make use of available resources wisely. Also, sometimes, to provide solutions to such problems, we need to think in terms of numbers.

Similarly, an operations research analyst analyses complex problems and develops mathematical models to provide alternate solutions for problems and helps to make better decisions.

Example 1: Find five rational numbers between 6 and 7 using the Average method.

Solution: A rational number between 6 and 7 is $\frac{6+7}{2} = \frac{13}{2}$

$\therefore\ 6 < \frac{13}{2} < 7$

A rational number between 6 and $\frac{13}{2} = \frac{6+\frac{13}{2}}{2} = \frac{25}{4}$

A rational number between $\frac{13}{2}$ and $7 = \frac{\frac{13}{2}+7}{2} = \frac{27}{4}$

A rational number between 6 and $\frac{25}{4} = \frac{\frac{25}{4}+6}{2} = \frac{49}{8}$

A rational number between $\frac{25}{4}$ and $7 = \frac{\frac{25}{4}+7}{2} = \frac{53}{8}$

$\therefore\ 6 < \frac{49}{8} < \frac{25}{4} < \frac{13}{2} < \frac{53}{8} < \frac{27}{4} < 7$

Hence, the required numbers are $\frac{49}{8}, \frac{25}{4}, \frac{13}{2}, \frac{53}{8}$ and $\frac{27}{4}$.

Both the methods are applicable for all rational numbers with the same as well as different denominators.

The mean of any two given numbers, a and $b = \frac{a+b}{2}$ (a and b being any rational numbers), helps to find rational numbers between the two given rational numbers.

Example 2: Find four rational numbers between $\frac{3}{2}$ and $\frac{7}{3}$ using both LCM and Average methods.

Solution: **LCM method:**

Here, the denominators of both the rational numbers are different. So, we will find the equivalents of the given rational numbers by finding the LCM of the denominators 2 and 3, which is 6.

The equivalent rational numbers of the given rational numbers that have the common denominator 6 are

$$\frac{3}{2}=\frac{3\times 3}{2\times 3}=\frac{9}{6} \text{ and } \frac{7\times 2}{3\times 2}=\frac{14}{6}$$

The rational numbers that lie between numbers that have the same denominator, i.e., $\frac{9}{6}$ and $\frac{14}{6}$ are $\frac{10}{6},\frac{11}{6},\frac{12}{6},\frac{13}{6}$ or $\frac{5}{3},\frac{11}{6},2,\frac{13}{6}$.

The rational numbers that lie between the given rational numbers are $\frac{5}{3},\frac{11}{6},2,\frac{13}{6}$.

Average method:

A rational number between $\frac{3}{2}$ and $\frac{7}{3}$ is $\dfrac{\frac{3}{2}+\frac{7}{3}}{2}=\frac{23}{12}$

$$\therefore \frac{3}{2}<\frac{23}{12}<\frac{7}{3}$$

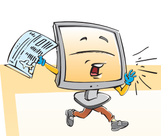
A rational number between $\frac{3}{2}$ and $\frac{23}{12}$ $=\dfrac{\frac{3}{2}+\frac{23}{12}}{2}=\frac{41}{24}$

A rational number between $\frac{23}{12}$ and $\frac{7}{3}$ $=\dfrac{\frac{23}{12}+\frac{7}{3}}{2}=\frac{51}{24}$

A rational number between $\frac{51}{24}$ and $\frac{7}{3}$ $=\dfrac{\frac{51}{24}+\frac{7}{3}}{2}=\frac{107}{48}$

$$\frac{3}{2}<\frac{23}{12}<\frac{41}{24}<\frac{51}{24}<\frac{107}{48}<\frac{7}{3}$$

Hence, the required numbers are $\frac{23}{12},\frac{41}{24},\frac{51}{24},\frac{107}{48}$.



Updates

The 'Rational Number Project' was funded by National Science Foundation (NSF). This project is a collection of more than 90 papers, chapters, several books and other project publications and is mainly concerned with the learning and teaching of rational number concepts which include fractions, decimals, ratio, etc.

Example 3: Find six rational numbers between $\frac{-1}{6}$ and $\frac{-2}{5}$ by LCM method and find their absolute values.

Solution: Here, the denominators of both the rational numbers are different. So, we will find the equivalents of the given rational numbers by finding the LCM of the denominators 6 and 5, which is 30.

The equivalent rational numbers of the given rational numbers that have the common denominator 30 are

$$\frac{-1}{6}\times\frac{5}{5}=\frac{-5}{30} \text{ and } \frac{-2}{5}\times\frac{6}{6}=\frac{-12}{30}$$

The rational numbers that lie between the given numbers that have the same denominator, i.e.,

$$\frac{-5}{30} \text{ and } \frac{-12}{30} \text{ are } \frac{-6}{30},\frac{-7}{30},\frac{-8}{30},\frac{-9}{30},\frac{-10}{30},\frac{-11}{30} \text{ or } \frac{-1}{5},\frac{-7}{30},\frac{-4}{15},\frac{-3}{10},\frac{-1}{3},\frac{-11}{30}.$$

Now, their absolute values are $\frac{6}{30},\frac{7}{30},\frac{8}{30},\frac{9}{30},\frac{10}{30},\frac{11}{30}$ or $\frac{1}{5},\frac{7}{30},\frac{4}{15},\frac{3}{10},\frac{1}{3},\frac{11}{30}$.

Example 4: Find thirteen rational numbers between $\frac{-2}{8}$ and $\frac{-3}{5}$.

Solution: Here, the denominators of both rational numbers are different, so we will find the equivalents of given rational numbers by finding the LCM of the denominators 5 and 8, which is 40.

The equivalent rational numbers of the given rational numbers that have the common denominator, i.e., 40 are

$$\frac{-2}{8}=\frac{-2\times5}{8\times5}=\frac{-10}{40} \text{ and } \frac{-3}{5}=\frac{-3\times8}{5\times8}=\frac{-24}{40}$$

The rational numbers that lie between the given rational numbers are

$$\frac{-11}{40},\frac{-12}{40},\frac{-13}{40},\frac{-14}{40},\frac{-15}{40},\frac{-16}{40},\frac{-17}{40},\frac{-18}{40},\frac{-19}{40},\frac{-20}{40},\frac{-21}{40},\frac{-22}{40},\frac{-23}{40}.$$

The rational numbers that lie between the given rational numbers are

$$\frac{-11}{40},\frac{-3}{10},\frac{-13}{40},\frac{-7}{20},\frac{-15}{40},\frac{-4}{5},\frac{-17}{40},\frac{-9}{20},\frac{-19}{40},\frac{-1}{2},\frac{-21}{40},\frac{-11}{20},\frac{-23}{40}.$$

The rational numbers, that lie between two given rational numbers and are found by using LCM method, should be converted into standard form.

Example 5: Find thirteen rational numbers between $\frac{-2}{5}$ and $\frac{-4}{5}$.

Solution: Here, the denominators of both rational numbers are the same and we have to find 13 rational numbers between the two given rational numbers. But looking at the difference between their numerators, i.e., –2 and –4, it is quite clear that there will be only 1 rational number that can be found out directly. Therefore, we will multiply both $\frac{-2}{5}$ and $\frac{-4}{5}$ with a bigger number so that the difference between –2 and –4 gets increased.

So, let us choose the number 7 with which the numerator and denominator of both the numbers, i.e., $\frac{-2}{5}$ and $\frac{-4}{5}$ are multiplied. So, we get,

$$\frac{-2}{5}=\frac{-2\times7}{5\times7}=\frac{-14}{35} \text{ and } \frac{-4}{5}=\frac{-4\times7}{5\times7}=\frac{-28}{35}$$

The rational numbers that lie between $\frac{-14}{35}$ and $\frac{-28}{35}$ are

$$\frac{-15}{35},\frac{-16}{35},\frac{-17}{35},\frac{-18}{35},\frac{-19}{35},\frac{-20}{35},\frac{-21}{35},\frac{-22}{35},\frac{-23}{35},\frac{-24}{35},\frac{-25}{35},\frac{-26}{35},\frac{-27}{35}.$$

∴ The rational numbers that lie between the given rational numbers in standard form are

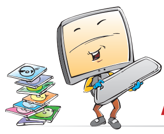
$$\frac{-3}{7},\frac{-16}{35},\frac{-17}{35},\frac{-18}{35},\frac{-19}{35},\frac{-4}{7},\frac{-3}{5},\frac{-22}{35},\frac{-23}{35},\frac{-24}{35},\frac{-5}{7},\frac{-26}{35},\frac{-27}{35}.$$

The more the number of rational numbers we need to find between the two given rational numbers (with the same denominator), the greater should be the number we choose to multiply the numerator and denominator of both the given rational numbers.

Processing—III

1. How many rational numbers can you find between $\frac{1}{2}$ and $\frac{1}{3}$?
2. Find a rational number that lies between 0 and 1.
3. Find 4 rational numbers that lie between –0.56 and –0.65.
4. Find any two rational numbers that lie between $\frac{1}{3}$ and $\frac{5}{7}$.
5. Does $\frac{15}{6}$ lie between 2 and 3? Can you find any other 4 rational numbers that lie between the two given numbers using LCM method and verify the answer by Average method.
6. Find any 5 rational numbers that lie between $\frac{491}{2000}$ and $\frac{491}{200}$ using LCM method. Also, find 10 rational numbers that are less than $\frac{491}{200}$.

The main advantage of LCM method over Average method is that it helps to find more number of rational numbers between two given rational numbers easily and in lesser time, especially when the given rational numbers have different denominators.



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Rational Numbers

Properties of Rational Numbers

Example to verify the properties of rational numbers under any of the operations (+, –, ×,÷)

1. Closure

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2. Commutative

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3. Associative

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4. Distributive

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Verification of the given properties for any rational number

1. Additive Identity

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2. Multiplicative Identity

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3. Additive Inverse

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4. Multiplicative Inverse

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Representation of Rational Numbers on the Number Line

1. Representation of a proper fraction on the number line

2. Representation of a improper fraction on the number line

3. Representation of a terminating decimal on the number line

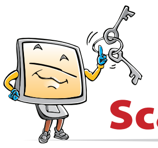
4. Representation of a non-terminating recurring decimal on the number line

Rational Numbers between Two Rational Numbers

Methods of finding rational numbers between two given rational numbers

1. LCM method

2. Average method



Scandisk

1. Name the properties of rational numbers used in the following.

   (a) $\frac{5}{7} \times 4 = 4 \times \frac{5}{7}$

   (b) $2 + \left(4 + \frac{-9}{11}\right) = (2 + 4) + \frac{-9}{11}$

   (c) $\frac{1}{2} \times \left(\frac{-1}{3} + \frac{2}{5}\right) = \left\{\frac{1}{2} \times \left(\frac{-1}{3}\right)\right\} + \frac{1}{2} \times \frac{2}{5}$

   (d) $\frac{1}{5} \times \left(\frac{2}{3} \times \frac{1}{7}\right) = \left(\frac{1}{5} \times \frac{2}{3}\right) \times \frac{1}{7}$

2. Choose the correct option.

   (a) Which is the multiplicative identity in the set of rational numbers?

   (i) –1 (ii) 0 (iii) 1

   (b) Reciprocal of 1 is

   (i) 1 (ii) –1 (iii) 0

3. Convert the following non-terminating recurring decimals into fractions.

   (a) $0.1\overline{43}$ (b) $1.3\overline{7}$ (c) $4.1\overline{13}$

4. What is the negative of the negative of a number? Explain with an example.
5. Which number does not have a reciprocal?
6. Find out a rational number that is equal to its reciprocal.
7. If the statements given below are true for all rational numbers, find x in each case.

   (a) $x \times \frac{-1}{9} \times \frac{2}{7} = \left\{\frac{-4}{5} \times \left(\frac{-1}{9}\right)\right\} \times \frac{2}{7}$

   (b) $\frac{21}{11} \times \left(\frac{1}{6} - x\right) = \frac{21}{11} \times \frac{1}{6} - \frac{21}{11} \times \frac{15}{4}$

   (c) $\frac{-14}{5} \div x = \frac{14}{5}$

   (d) $\frac{16}{3} \times \left(\frac{-1}{5}\right) = x \times \frac{16}{3}$

8. Is $\frac{3}{13}$ the multiplicative inverse of $-4\frac{1}{3}$? Give reasons for your answer.
9. The sum of two rational numbers is $\frac{11}{5}$. If one of them is $\frac{-2}{3}$, find the other.
10. Vriti had ₹ 600. She spent $\frac{2}{3}$ of her total money on buying a t-shirt and $\frac{1}{5}$ of the remaining money on buying hair clips. How much money is left with her?
11. Verify that $-(-x) = x$ for:

    (a) $x = \frac{21}{2}$

    (b) $x = \frac{-14}{5}$

12. Verify associativity of a multiplication of rational numbers for each of the following pairs of rational numbers:

    (a) $\frac{17}{3}$ and $\frac{-8}{5}$ (b) $\frac{-33}{7}$ and $\frac{-12}{7}$

13. Verify $|a| = a$ and $|-a| = a$ for

    (a) $a = \frac{-4}{7}$ (b) $a = \frac{5}{3}$

14. Verify the given expressions, if $X = \frac{1}{3}$, $Y = \frac{-2}{5}$, $Z = \frac{3}{4}$.

    (a) $X + Y = Y + X$ (b) $X + (Y + Z) = (X + Y) + Z$ (c) $X + 0 = X$

    (d) $X - 0 = X$ (e) $X \times Y = Y \times X$ (f) $X \times (Y \times Z) = (X \times Y) \times Z$

    (g) $X \times (Y + Z) = X \times Y + X \times Z$ (h) $X \times (Y - Z) = X \times Y - X \times Z$

15. Using properties of rational numbers, evaluate $\frac{2}{5} \times \left(\frac{-3}{7}\right) - \frac{1}{6} \times \frac{3}{2} + \frac{1}{14} \times \frac{2}{5}$.

16. Plot the following on the number line.

    (a) $\frac{-3}{4}$ (b) $\frac{5}{8}$ (c) $\frac{10}{3}$ (d) $\frac{-17}{6}$ (e) -1.7

    (f) $\frac{-2}{3}$ (g) 5.3 (h) $-1.\overline{2}$ (i) $2.\overline{33}$

17. Represent $\frac{-2}{5}$ and 0.5 on the number line and also insert 5 rational numbers between them.

18. Find two rational numbers that lie between the following sets of numbers.

    (a) -5.76 and -5.77 (b) $2\frac{1}{3}$ and $3\frac{5}{7}$ (c) -6 and $\frac{3}{5}$

19. Solve and plot the following on the number line.

    (a) $\frac{4}{3} \times \frac{1}{2} \div \frac{3}{2}$ (b) $\frac{21}{2} \times \frac{1}{7} \div 3 - \frac{4}{5}$

Explorer

Do you know the people who are behind organising your favourite sports? They are the Sports Events Organisers who organise, provide funds and facilitate for the sports events.

Have you organised a sports event in your school? For this, you need to collect all the relevant information about the event.

You need to analyse and interpret the given data about different sports and perform calculations.

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Sports Events Organiser

Organises funds, facilities and promotional activities for sports events and deals with sportspersons as well as large audiences

Should I get Dhoni or Andrew Symonds for the next event?

Operations Research Analyst

Analyses complex problems, formulates and applies mathematical methods to develop and interpret information and helps to solve problems and make better decisions

Need help for planning error free processes? I'm the person for you!

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- Rational Numbers
- Rational Numbers on The Number Line And Their Order Relationship
- Rational Numbers SE 2
- Rational Numbers SE 4
- The Rational Number Line